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Duality and $4d$ String Dynamics[1]

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Abstract

We review some examples of heterotic/type II string duality which shed light on the infrared dynamics of string compactifications with N=2 and N=1 supersymmetry in four dimensions.

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1 Introduction

I am very happy to have the opportunity to speak about strong/weak coupling duality on this occasion honoring the 60th birthday of Professor Keiji Kikkawa. His own foundational work on T-duality [1], the worldsheet analogue of S-duality, was in many ways instrumental in inspiring the recent developments in nonperturbative string theory.

Strong-weak coupling dualities now allow us to determine the strong coupling dynamics of string vacua with $N \geq 4$ supersymmetry in four dimensions [2]. It is natural to ask if this progress in our understanding of string theory can be extended to the more physical vacua with less supersymmetry. For N=2 theories in four dimensions, quantum corrections significantly modify the mathematical structure of the moduli space of vacua, as well as the physical interpretation of its apparent singularities. This was beautifully demonstrated in the field theory case in [3] and it has more recently become possible to compute the exact quantum moduli spaces for N=2 string compactifications as well [4, 5]. This constitutes the subject of the first part of my talk.

Of course, the case of most physical interest is $N \leq 1$ theories. In the second part of my talk, I discuss examples of dual heterotic/type II string pairs where the heterotic theory is expected to exhibit nonperturbative dynamics which may fix the dilaton and break supersymmetry [6]. The type II dual manages to reproduce the qualitative features expected of the heterotic side at tree level. It is to be hoped that further work along similar lines will result in a better understanding of supersymmetry breaking in string theory.

The first part of this talk is based on joint work with C. Vafa, and the second part of this talk is based on joint work with E. Silverstein.

2 N=2 Gauge Theory and String Compactifications

Recall that the N=2 gauge theory with gauge group $SU(2)$ is the theory of a single N=2 vector multiplet consisting of a vector A^{μ}, two Weyl fermions λ and ψ, and a complex scalar field ϕ, all in the adjoint representation of $SU(2)$. In N=1 language, this is a theory of an N=1 vector multiplet (λ, A^{μ}) coupled to an N=1 chiral multiplet (ϕ, ψ). The scalar potential of the theory is determined by supersymmetry to be

$$V(\phi) = \frac{1}{g^2}[\phi, \phi^{+}]^2 \tag{1}$$

We see that V vanishes as long as we take $\phi = diag(a, -a)$, so there is a moduli space of classical vacua parameterized by the gauge invariant parameter $u = tr(\phi^2)$.

At generic points in this moduli space $\mathcal{M}_v$ of vacua, there is a massless N=2 U(1) vector multiplet A. The leading terms in its effective lagrangian are completely determined in terms of a single holomorphic function $F(A)$, the prepotential:

$$L \sim \int d^4\theta \frac{\partial F}{\partial A}\bar{A} + \int d^2\theta \frac{\partial^2 F}{\partial A^2} W_\alpha W^\alpha + c.c. \tag{2}$$

The first term determines, in N=1 language, the Kahler potential (and hence the metric on $\mathcal{M}_v$) while the second term determines the gauge coupling as a function of moduli.

In [3] the exact form of F including instanton corrections was determined. In addition, the masses of all of the BPS saturated particles were computed. This was reviewed in great detail in several other talks at this conference, so I will not repeat the solution here. It will suffice to say that the crucial insight is that the singular point $u = tr(\phi^2) = 0$ where $SU(2)$ gauge symmetry is restored in the classical theory splits, in the quantum theory, into two singular points $u = \pm\Lambda^2$, where a monopole and a dyon become massless.

In this talk our interest is not really in $N = 2$ gauge theories but in the string theories which reduce to $N = 2$ gauge theories in the infrared. There are two particularly simple classes of $d = 4, N = 2$ supersymmetric string compactifications. One obtains such theories from Type II (A or B) strings on Calabi-Yau manifolds, and from heterotic strings on $K_3 \times T^2$ (with appropriate choices of instantons on the K_3). Here we briefly summarize some basic properties of these theories.

Type IIA strings on a Calabi-Yau threefold M give rise to a four-dimensional effective theory with n_v vector multiplets and n_h hypermultiplets where

$$n_v = h^{1,1}(M), \quad n_h = h^{2,1}(M) + 1 \tag{3}$$

The $+1$ in n_h corresponds to the fact that for such type II string compactifications, the *dilaton* is in a hypermultiplet.

The vector fields in such a theory are Ramond-Ramond U(1)s, so there are no charged states in the perturbative string spectrum. Furthermore, because of the theorem of de Wit, Lauwers, and Van Proeyen [7] which forbids couplings of vector multiplets to neutral hypermultiplets in N=2 effective lagrangians, the dilaton does not couple to the vector moduli. This means that there are no perturbative or nonperturbative corrections to the moduli space of vector multiplets. On the other hand the moduli spaces of hypermultiplets are expected to receive highly nontrivial corrections, including "stringy" corrections with $e^{-1/g}$ strength [8].

One interesting feature of the moduli spaces of vector multiplets in such theories is the existence of conifold points at finite distance in the moduli space. At

such points the low energy effective theory becomes singular (e.g., the prepotential develops a logarithmic singularity) [9]. This phenomenon is reminiscent of the singularities in the prepotential which occur at the "massless monopole" points in the Seiberg-Witten solution of N=2 gauge theory, singularities which are only present because one has integrated out a charged field which is becoming massless. In the case at hand, in fact, one can show that there are BPS saturated states (obtained by wrapping 2-branes around collapsing 2-cycles) which become massless and which are charged under (some of) the Ramond-Ramond $U(1)$s [10]. These explain the singularity in the prepotential. In fact at special such points, where enough charged fields (charged under few enough $U(1)$s) become massless, one can give them VEVs consistent with D and F flatness. This results in new "Higgs branches" of the moduli space. These new branches correspond to string compactifications on different Calabi-Yau manifolds, topologically distinct from M [11], and there is evidence that all Calabi-Yau compactifications may be connected in this manner [12, 13].

The other simple way of obtaining an N=2 theory in four dimensions from string theory is to compactify the heterotic string (say $E_8 \times E_8$) on $K_3 \times T^2$. Because of the Bianchi identity

$$dH = Tr(R \wedge R) - Tr(F \wedge F) \tag{4}$$

one must embed 24 instantons in the $E_8 \times E_8$ in order to obtain a consistent theory. An $SU(N)$ k-instanton on K_3 comes with $Nk + 1 - N^2$ hypermultiplet moduli (where $k \geq 2N$), and K_3 comes with 20 hypermultiplet moduli which determine its size and shape. Embedding an $SU(N)$ instanton in E_8 breaks the observable low energy gauge group to the maximal subgroup of E_8 which commutes with $SU(N)$ (E_7 for N=2, E_6 for N=3, and so forth).

In addition, there are three $U(1)$ vector multiplets associated with the T^2. Their scalar components are the dilaton S and the complex and kahler moduli τ and ρ of the torus (both of which live on the upper half-plane H mod $SL(2, Z)$). At special points in the moduli space the $U(1)^2$ associated with τ and ρ is enhanced to a nonabelian gauge group:

$$\tau = \rho \rightarrow SU(2) \times U(1), \quad \tau = \rho = i \rightarrow SU(2)^2, \quad \tau = \rho = 1/2 + i\sqrt{3}/2 \rightarrow SU(3) \tag{5}$$

Because the dilaton lives in a vector multiplet in such compactifications, the moduli space of vectors is modified by quantum effects. On the other hand, the moduli space of hypermultiplets receives neither perturbative nor nonperturbative corrections.

An interesting feature of the heterotic $\mathcal{M}_v$ is the existence of special points where the classical theory exhibits an enhanced gauge symmetry (as described above for the compactification on T^2). Sometimes by appropriate passage to a Higgs or Coulomb phase, such enhanced gauge symmetry points link moduli spaces of N=2

heterotic theories which have different generic spectra (for some examples see [4, 14]). It is natural to conjecture that such transitions connect all heterotic N=2 models, in much the same way that conifold transitions connect Calabi-Yau compactifications of type II strings.

3 N=2 String-String Duality

¿From the brief description of heterotic and type II N=2 vacua in the previous section, it is clear that a duality relating the two classes of theories would be extremely powerful. If one were to find a model with dual descriptions as a compactification of the Type IIA string on M and the heterotic string on $K_3 \times T^2$, one could compute the exact prepotential for $\mathcal{M}_v$ from the Type IIA side (summing up what from the heterotic perspective would be an infinite series of instanton corrections). Similarly, one would get exact results for $\mathcal{M}_h$ from the heterotic side – this would effectively compute the $e^{-1/g}$ corrections expected from the IIA perspective. In fact, such a duality has been found to occur in several examples in [4, 5].

One of the simplest examples is as follows. Consider the heterotic string compactified to eight dimensions on T^2 with $\tau = \rho$. Further compactify on a K_3, satisfying the Bianchi identity for the H field by embedding $c_2 = 10$ $SU(2)$ instantons in each E_8 and a $c_2 = 4$ $SU(2)$ instanton into the "enhanced" $SU(2)$ arising from the $\tau = \rho$ torus. After Higgsing the remaining E_7 gauge groups one is left with a generic spectrum of 129 hypermultiplets and 2 vector multiplets. The 2 vectors are τ and the dilaton S – when $\tau = i$, one expects an $SU(2)$ gauge symmetry to appear (the other $SU(2)$ factor that would normally appear there has been broken in the compactification process).

This tells us that if there is a type IIA dual compactification on a Calabi-Yau M, then the Betti numbers of M must be

$$h_{11}(M) = 2, \quad h_{21}(M) = 128 \tag{6}$$

There is a known candidate manifold with these Betti numbers – the degree 12 hypersurface in $WP^4_{1,1,2,2,6}$ defined by the vanishing of p

$$p = z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 + ... \tag{7}$$

This manifold has in fact been studied intensively as a simple example of mirror symmetry in [15, 16].

The mirror manifold W has $h_{11}(W) = 128, h_{21}(W) = 2$. The conjecture that IIA on M is equivalent to the heterotic string described above implies that IIB on W is also equivalent to that heterotic string. The structure of the moduli space of vector multiplets of the heterotic string should be *exactly* given by the classical (in both

sigma model and string perturbation theory) moduli space of complex structures of W.

The mirror manifold can be obtained by orbifolding $p = 0$ by the maximal group of phase symmetries which preserves the holomorphic three-form [17]. Then the two vector moduli are represented by ψ and ϕ in the polynomial

$$p = z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^6 z_2^6 \tag{8}$$

It is also useful, following [15], to introduce

$$x = \frac{-1}{864}\frac{\phi}{\psi^6}, \quad y = \frac{1}{\phi^2} \tag{9}$$

These are the convenient "large complex structure" coordinates on the moduli space of vector multiplets for the IIB string.

In order to test our duality conjecture, we should start by checking that the IIB string reproduces some qualitative features that we expect of the heterotic $\mathcal{M}_v$. For example, $\tau = i$ for weak coupling $S \to \infty$ is an $SU(2)$ point. There should therefore be a singularity of $\mathcal{M}_v$ at this point which splits, as one turns on the string coupling, to *two* singular points (where monopoles/dyons become massless), as in the case of pure $SU(2)$ gauge theory.

The "discriminant locus" where the IIB model becomes singular is given by

$$(1 - x)^2 - x^2 y = 0 \tag{10}$$

So we see that as a function of y for $y \neq 0$ there are two solutions for x and as $y \to 0$ they merge to a single singular point $x = 1$. This encourages us to identify $x = 1, y = 0$ with $\tau = i, S \to \infty$ of the heterotic string – the $SU(2)$ point. The metric on the moduli space for y at $y = 0$ and S at weak coupling also agree if one makes the identification $y \sim e^{-S}$.

There is also a remarkable observation in [16] that the mirror map, restricted to $y = 0$, is given by

$$x = \frac{j(i)}{j(\tau_1)} \tag{11}$$

where τ_1 is one of the coordinates on the Kahler cone of M. Here j is the elliptic j-function mapping C onto $H/SL(2, Z)$. This tells us that the classical heterotic τ moduli space, which is precisely $H/SL(2, Z)$, is embedded in the moduli space of M at weak coupling precisely as expected from duality. In fact using the uniqueness of special coordinates up to rotations, one can find the exact formula expressing the IIB coordinates (x, y) in terms of the heterotic coordinates (τ, S).

Of course with this map in hand there are now several additional things one can check. The tests which have been performed in [4, 18, 19, 20] include

1) A matching of the expected loop corrections to the heterotic prepotential with the form of the tree-level exact Calabi-Yau prepotential.

2) A test that the g-loop F-terms computed by the topological partition functions F_g on the type II side (which include e.g. R^2 and other higher derivative terms) are reproduced by appropriate (one-loop!) computations on the heterotic side.

3) A demonstration that in an appropriate double-scaling limit, approaching the $\tau = i$, $S \to \infty$ point of the heterotic string while taking $\alpha^{'} \to 0$, the IIB prepotential reproduces the exact prepotential of $SU(2)$ gauge theory (including Yang-Mills instanton effects) computed in [3].

These tests give very strong evidence in favor of the conjectured duality. Given its veracity, what new physics does the duality bring into reach?

• One now has examples of four-dimensional theories with exactly computable quantum gravity corrections. In the example discussed above, the Seiberg-Witten prepotential which one finds in an expansion about $\tau = i, S \to \infty$ receives gravitational corrections which are precisely computable as a power series in $\alpha^{'}$.

• On a more conceptual level, the approximate duality of [3] between a microscopic $SU(2)$ theory (at certain points in its moduli space) and a $U(1)$ monopole/dyon theory is promoted to an *exact* duality, valid at all wavelengths, between heterotic and type II strings.

• There is evidence that at strong heterotic coupling, new gauge bosons and charged matter fields appear, sometimes giving rise to new branches of the moduli space [21, 22].

• The $e^{-1/g}$ corrections to the hypermultiplet moduli space of type II strings are in principle exactly computable using duality (and may be of some mathematical interest).

One might wonder what is special about the Calabi-Yau manifolds which are dual to weakly coupled heterotic strings. In fact it was soon realized that the examples of duality in [4] involve Calabi-Yau manifolds which are K_3 fibrations [23]. That is, locally the Calabi-Yau looks like $CP^1 \times K_3$. In fact, one can prove that if the type IIA string on a Calabi-Yau M (at large radius) is dual to a weakly coupled heterotic string, then M must be a K_3 fibration [24].

To make this more concrete, in the example of the previous section, we saw M was defined by the vanishing of

$$p = z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 + ... \qquad (12)$$

in $WP^4_{1,1,2,2,6}$. Set $z_1 = \lambda z_2$ and define $y = z_1^2$ (which is an allowed change of variables since an identification on the WP^4 takes $z_1 \to -z_1$ without acting on $z_{3,4,5}$). Then

the polynomial becomes (after suitably rescaling to absorb λ)

$$p = y^6 + z_3^6 + z_4^6 + z_5^2 + ... \tag{13}$$

which defines a K_3 surfaces in $WCP^3_{1,1,1,3}$. The choice of λ in $z_1 = \lambda z_2$ is a point on CP^1, and the K_3 for fixed choice of λ is the fiber.

It is not surprising that K_3 fibrations play a special role in 4d N=2 heterotic/type II duality. Indeed the most famous example of heterotic/type II duality is the 6d duality between heterotic strings on T^4 and type IIA strings on K_3 [25, 26]. If one compactifies the type IIA string on a CY threefold which is a K_3 fibration, and simultaneously compactifies the heterotic string on a $K_3 \times T^2$ where the K_3 is an elliptic fibration, then locally one can imagine taking the bases of both fibrations to be large and obtaining in six dimensions an example of the well-understood 6d string-string duality [27]. This picture is not quite precise because of the singularities in the K_3 fibration, but it does provide an intuitive understanding of the special role of K_3 fibrations.

4 N=1 Duality and Gaugino Condensation

Starting with an $N = 2$ dual pair of the sort discussed above, one can try to obtain an $N = 1$ dual pair by orbifolding both sides by freely acting symmetries. This strategy was used in [27, 28] where several examples with trivial infrared dynamics were obtained. Here we will find that examples with highly nontrivial infrared dynamics can also be constructed [6].

Our starting point is an N=2 dual pair (IIA on a Calabi-Yau M and heterotic on $K_3 \times T^2$) where the heterotic gauge group takes the form

$$G \;=\; E_8^H \otimes E_7^{obs} \otimes ... \tag{14}$$

H denotes the hidden sector and *obs* the observable sector. We will first discuss the technical details of the Z_2 symmetry by which we can orbifold both sides to obtain an $N = 1$ dual pair, and then we discuss the physics of the duality.

Orbifold the heterotic side by the Enriques involution acting on K_3 and a total reflection on the T^2. This acts on the base of the elliptic fibration (z_1, z_2) by

$$(z_1, z_2) \;\rightarrow\; (\bar{z}_2, -\bar{z}_1) \tag{15}$$

taking $CP^1 \rightarrow RP^2$. In addition, we need to choose a lifting of the orbifold group to the gauge degrees of freedom.

We do this as follows:

• Put a modular invariant embedding into the "observable" part of the gauge group alone.
• Embed the translations which generate the T^2 into E_8^H, constrained by maintaining level-matching and the relations of the space group. For example one could take Wilson lines $A_{1,2}$ along the a and b cycle of the T^2 given by

$$A_1 = \frac{1}{2}(0,0,0,0,1,1,1,1), \quad A_2 = \frac{1}{2}(-2,0,0,0,0,0,0,0) \tag{16}$$

Here $A_{1,2} = \frac{1}{2}L_{1,2}$ where $L_{1,2}$ are vectors in the E_8 root lattice. These Wilson lines break the E_8^H gauge symmetry to $SO(8)_1 \otimes SO(8)_2$.

How does the Z_2 map over to the type II side? ¿From the action

$$(z_1, z_2) \;\rightarrow\; (\bar{z}_2, -\bar{z}_1) \tag{17}$$

on the CP^1 base (which is common to both the heterotic and type II sides), we infer that the Z_2 must be an antiholomorphic, orientation-reversing symmetry of the Calabi-Yau manifold M. To make this a symmetry of the type IIA string theory, we must simultaneously flip the worldsheet orientation, giving us an "orientifold." In such a string theory, one only includes maps Φ of the worldsheet Σ to spacetime M/Z_2 if they satisfy

$$\Phi^*(w_1(M/Z_2)) = w_1(\Sigma) \tag{18}$$

where w_1 is the first Stieffel-Whitney class.

We know from 6d string-string duality that the Narain lattice $\Gamma^{20,4}$ of heterotic string compactification on T^4 maps to the integral cohomology lattice of the dual K_3. This means that we can infer from the action of the Z_2 on the heterotic gauge degrees of freedom, what the action of the Z_2 must be on the integral cohomology of the K_3 fiber on the IIA side. Since we are frozen on the heterotic side at a point with $SO(8)^2$ gauge symmetry in the hidden sector, the dual K_3 must be frozen at its singular enhanced gauge symmetry locus.

The K_3 dual to heterotic enhanced gauge symmetry G has rational curves C_i, $i = 1, ..., rank(G)$ shrinking to zero area (with the associated $\theta_i = 0$ too). It is easy to see, e.g. from Witten's gauged linear sigma model that in this situation the type II theory indeed exhibits an extra Z_2 symmetry. The bosonic potential of the relevant gauged linear sigma model (for the case of a single shrinking curve) is given by

$$V = \frac{1}{2e^2}\sum_i \Big\{ \Big([\sum_\alpha Q_i^\alpha (|\phi_\alpha^i|^2 - |\tilde{\phi}_\alpha^i|^2)] - r_i^0 \Big)^2$$

$$+ \Big(Re(\sum_\alpha \phi_\alpha^i \tilde{\phi}_\alpha^i) - r_i^1 \Big)^2 + \Big(Im(\sum_\alpha \phi_\alpha^i \tilde{\phi}_\alpha^i) - r_i^2 \Big)^2 \Big\}$$

$$+\frac{1}{2}\sum_i [\sum_\alpha Q_i^{\alpha\ 2}(|\phi_\alpha^i|^2 + |\tilde{\phi}_\alpha^i|^2)]|\sigma_i|^2$$

Here the ϕs represent the K_3 coordinates while r parametrizes the size of the curve and σ is the Kahler modulus. Precisely when $\vec{r} \to 0$, the model has the Z_2 symmetry $\phi \to -\tilde{\phi}$, $\sigma \to -\sigma$. Orbifolding by this Z_2 then freezes the K_3 at its enhanced gauge symmetry locus, as expected.

What is the physics of the dual pairs that one constructs in this manner? In the heterotic string, when there is a hidden sector pure gauge group

$$G^{hidden} = \Pi\ G^b \tag{19}$$

one expects gaugino condensation to occur. This induces an effective superpotential

$$W = \sum\ h_b\ \Lambda_b^3(S) \tag{20}$$

where $\Lambda_b(S) \sim e^{-\alpha_b S}$ and α_b is related to the beta function for the running G_b coupling. It was realized early on [29, 30] that in such models (with more than one hidden factor) one might expect both stabilization of the dilaton and supersymmetry breaking. It has remained a formidable problem to determine which (if any) such models actually do have a stable minimum at weak coupling with broken supersymmetry.

For now, we will be content to simply understand how the *qualitative* structure of the heterotic theory (e.g. the gaugino-condensation induced effective superpotential) is reproduced by the type II side. This is mysterious because the type II N=2 theory we orientifolded had only abelian gauge fields in its spectrum, so we need to reproduce the strongly coupled nonabelian dynamics of the heterotic string with an *abelian* gauge theory on the type II side.

The heterotic orbifold indicates the spectrum of the string theory as $g_{het} \to 0$. The heterotic dilaton S maps to the radius R of the RP^2 base of the type II orientifold (recall one obtains the RP^2 by orbifolding the base P^1 of the K_3 fibration)

$$S_{het} \leftrightarrow R_{RP^2} \tag{21}$$

The purported stable vacuum of the heterotic theory should then be expected to lie at large radius for the base, and on the (orientifold of the) conifold locus dual to enhanced gauge symmetry. There are two crucial features of this locus:

1) The RP^2 base has $\pi_1(RP^2) = Z_2$. So a state projected out in orientifolding the N=2 theory will have a massive version invariant under the Z_2. Say $\beta \in \pi_1(RP^2)$ is the nontrivial element. Take x a coordinate along an appropriate representative of β – a representative can be obtained by taking the image of a great circle on the original base P^1 after orientifolding. Then if the original non-invariant vertex operator was V, a new invariant vertex operator is given adiabatically by

$$V' = e^{\frac{ix}{R}}V \tag{22}$$

The Z_2 takes x to $x + \pi R$ and therefore V' is invariant if V was not. In particular this gives us massive versions of the scalars $a^i_{b,D}$ in the N=2 vector multiplets for G^b with masses

$$M_a \sim \frac{1}{R^2} \tag{23}$$

Effectively, for very large R, one is restoring the original N=2 supersymmetry.

2) The low energy theory for IIA at the conifold locus contains massless *solitonic* states [10]. One can see that they survive the N=2 $\rightarrow$ N=1 orientifolding by examining the behavior of the gauge couplings [27]. These extra solitonic states play the role of the "monopole hypermultiplets" M^b_i, $\tilde{M}^b_i$ of the N=2 theory.

These two facts taken together imply that as $R \rightarrow \infty$ there is an effective superpotential

$$W_{II} = \sum_b \left(m_b u^b_2(a^i_{b,D}, R) + \sum_{i=1}^{rank(b)} M^b_i a^i_{b,D} \tilde{M}^b_i \right) \tag{24}$$

where u^b_2 is the precise analogue of u of §2 for G^b and its functional dependence on R can be found from the $N = 2$ dual pair. As we'll now discuss, this structure

a) Allows us to reproduce the gaugino-condensation induced effective superpotential of the heterotic side.

b) Implies $\langle M \rangle \neq 0$, suggesting a geometrical description of the type II side by analogy with N=2 conifold transitions.

To see a), recall how the physics of N=1 $SU(2)$ gauge theory is recovered from the N=2 theory in [3]. One can obtain the N=1 theory by giving a bare mass to the adjoint scalar in the N=2 vector multiplet and integrating it out. In the vicinity of the monopole points this means there is an effective superpotential

$$W = mu(a_D) + \sqrt{2} a_D M \tilde{M} \tag{25}$$

Using the equations of motion and D-flatness, one finds

$$|\langle M \rangle| = |\langle \tilde{M} \rangle| = (-mu'(0)/\sqrt{2})^{1/2}, \quad a_D = 0 \tag{26}$$

The monopoles condense and given a mass to the (dual) $U(1)$ gauge field by the Higgs mechanism, leaving a mass gap. Two vacua arise in this way – one at each of the monopole/dyon points – in agreement with the Witten index computation for pure $SU(2)$ gauge theory.

In our case, we expect that condensation of the massless solitons will lead to the gaugino condensation induced superpotential (and, perhaps, supersymmetry breaking). To see this we must expand W_{II} in $a^i_{b,D}$ in anticipation of finding a minimum

at small a_D (since the minimum was at $a_D = 0$ in the global case). Using

$$u_2(a^i_{b,D}, R)) = e^{i\gamma^b}\lambda_b^2(R) + \cdots \tag{27}$$

(where γ^b comes from the phase of the gaugino condensate) as well as the matching condition

$$m_b\Lambda^2_{b,high} = \Lambda^3_{b,low} \tag{28}$$

one obtains from integrating out the massive adjoint scalar, one sees

$$W_{II} = \sum_b e^{i\gamma^b}\Lambda_b^3(S) + \cdots \tag{29}$$

Simply minimizing the supergravity scalar potential

$$V = e^K(D_iWG^{i\bar{j}}D_{\bar{j}}W - 3|W|^2) + \frac{1}{2}g^2D^2 \tag{30}$$

we also find that

$$\langle M_i^b\tilde{M}_i^b\rangle = -h_bm_bu^b_{2,i}(S) - K_iW \tag{31}$$

That is, the "wrapped two-branes" which give us the massless monopoles have condensed, in accord with the global result. So integrating out the massive $M, \tilde{M}$ and adjoint scalar degrees of freedom yields the same form of bosonic potential that we expect from gaugino condensation on the heterotic side.

In summary, we have argued that the type II dual description of the effects of gaugino condensation involves a mass perturbation breaking N=2 supersymmetry. One cannot add mass terms by hand in string theory: The type II orientifold produces the requisite massive mode as a Kaluza-Klein excitation of the original N=2 degrees of freedom that were projected out.

One intriguing feature of the IIA vacuum is the nonzero VEVs for the wrapped two-branes $M, \tilde{M}$. In the N=2 context $\langle M\rangle \neq 0 \rightarrow$ conifold transition. There is a well known geometrical description of the conifold points. For example, in the IIB theory the conifold in vector multiplet moduli space is obtained by going to a point in $\mathcal{M}_{complex}$ where there is a cone over $S^3 \times S^2$ in the Calabi-Yau. One can either "deform the complex structure" (return to the Coulomb phase, in physics language) by deforming the tip of the cone into an S^3, or one can do a "small resolution" and blow the tip of the cone into an S^2. The latter corresponds to moving to a new Higgs phase, in the N=2 examples [11].

It was noted long ago by Candelas, De La Ossa, Green, and Parkes [9] that at a *generic* conifold singularity such a small resolution does not produce a Kahler manifold. They speculated that such nonKahler resolutions might correspond to supersymmetry breaking directions. It is natural to suggest that we might be seeing a realization of that idea by duality. The analogy with N=2 conifold transitions suggests that $\langle M\rangle \neq 0 \rightarrow$ nonKahler resolution. One can hope that this will provide a useful dual view of supersymmetry breaking in string theory.

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